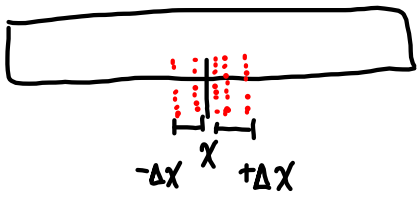
Absolute Uncertainty

The absolute uncertainty of a value is an interval above or below that value such that measurements of the value all lie within this interval.

It is the magnitude of the maximum difference between the value (or an estimation of the value) and a measurement of that value.



$x \pm \Delta x$

value uncertainty (absolute)

Relative Uncertainty (fractional Uncertainty)

Consider an absolute uncertainty of 1m (Δx):

very precise

If $x = 1008\text{m}$ and $\Delta x = 1\text{m} \Rightarrow (1008 \pm 1)\text{m}$

value absolute uncertainty

not very precise

If $x = 5\text{m}$ and $\Delta x = 1\text{m} \Rightarrow (5 \pm 1)\text{m}$

The absolute uncertainty does not necessarily give a good indication as to the precision of the measurement.

The quality of the measurement is better indicated by relative uncertainty by using the ratio:

$$\frac{\Delta x}{x}$$

(or fractional)

← The relative uncertainty is the ratio of the absolute uncertainty to the measurement itself.

Percentage Uncertainty

The relative uncertainty expressed as a percentage.

Summary:

measurement → x
absolute uncertainty ⇒ Δx
} $x \pm \Delta x$

relative uncertainty ⇒ $\frac{\Delta x}{x}$

percentage uncertainty ⇒ $\frac{\Delta x}{x} \cdot 100\%$

Example: The weight of an object is measured to be 2.7N with an absolute uncertainty of 0.1N.

measurement → 2.7N
absolute uncertainty → 0.1N
} (2.7 ± 0.1)N

relative uncertainty → $\frac{0.1\text{N}}{2.7\text{N}} = 0.04$ ↑ 1 sf

percentage uncertainty → $0.04(100\%) = 4\%$

Example: A length of 10m and a length of 10mm are each measured with an absolute uncertainty of 2mm. What is the relative uncertainty and percentage uncertainty for each? Which is more precise?

write both as metres (same place value)

$10\text{m} \pm 2\text{mm}$ (abs unc)

(10.000 ± 0.002)m

rel unc: $\frac{0.002\text{m}}{10.000\text{m}} = 0.0002$

perc unc: $0.0002 \times 100\% = 0.02\%$ $(2 \times 10^{-2}\%)$

MORE PRECISE ↗

$10\text{mm} \pm 2\text{mm}$

(10 ± 2)mm

$\frac{2\text{mm}}{10\text{mm}} = 0.2$

$0.2 \times 100\% = 20\%$

Adding and Subtracting

When adding or subtracting values, add the absolute uncertainties to give the absolute uncertainty in the result.

If $y = a \pm b$ then: $\Delta y = \Delta a + \Delta b$

$\pm$ for adding or subtracting

You always add the absolute uncertainties

$y = a + b$
or $y = a - b$

Example:

$(9.7 \pm 0.5)\,m + (4.3 \pm 0.2)\,m = (14.0 \pm 0.7)\,m$

$(9.7 \pm 0.5)\,m - (4.3 \pm 0.2)\,m = (5.4 \pm 0.7)\,m$

Example: Determine the perimeter of a square of side (2.4 ± 0.5) cm

$$\begin{array}{rl} & 2.4 \pm 0.5 \text{ cm} \\ & 2.4 \pm 0.5 \text{ cm} \\ & 2.4 \pm 0.5 \text{ cm} \\ + & 2.4 \pm 0.5 \text{ cm} \\ \hline & 9.6 \pm 2.0 \text{ cm} \end{array}$$

(10 ± 2) cm

or

$4\,(2.4 \pm 0.5)$ cm

(9.6 ± 2.0) cm

(10 ± 2) cm